Entry and Quality Competition in Hotelling Model with Exogenous Prices

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ABSTRACT

We examine a variant of Hotelling model where prices and locations are fixed but quality is a choice variable. We show that when sellers of a horizontally differentiated product are price-constrained and compete in quality, entry can lead to degradation of quality offered in equilibrium and reduction of consumers' surplus. Our findings may have policy implications for industries such as telecommunications where price is regulated and both horizontal and vertical differentiation is important.

Key words: Horizontal differentiation, Vertical differentiation, Entry

JEL Classification: D40, L10

I. INTRODUCTION

Hotelling’s (1929) linear city is a well-known model of horizontal differentiation where sellers’ choices of prices and locations (extent of differentiation) are discussed. This paper explores a non-price (‘quality’) dimension of competition in Hotelling model where both prices and locations are fixed exogenously. In telecommunications industry, for example, prices are regulated and remain stable.

* First received, October 19, 2012; Revision received, December 18, 2012; Accepted, February 2, 2013.
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over an extended period while sellers engage in acts of differentiation, both horizontal and vertical. We are going to show that when prices are exogenously given and sellers are horizontally differentiated, a new entry into a monopolistic market may lead to quality degradation and reduction of consumers’ surplus.

Suppose consumers are uniformly distributed on the unit interval \([0,1]\). Each consumer decides whether to purchase one unit of a product. When a consumer located at \(x \in [0,1]\) contemplates purchasing from a seller located at \(z \in [0,1]\), the consumer faces three parameters \(s, p\) and \(t\) that determine her net utility \(s - p - t|x - z|\). Each consumer’s reservation utility is normalized to be zero, so she purchases a product only if \(s - p - t|x - z| \geq 0\).

The parameter \(s\) represents a consumer’s gross surplus obtained from consuming the product and we refer to it as the product’s “quality” hereafter. The parameter \(p\) is the mill price charged by the seller. The parameter \(t\) is the so-called transportation cost per distance \(|x - z|\), where the distance may be interpreted in terms of geographical or any other dimensions of horizontal differentiation. \(^1\)

In Hotelling’s (1929) original analysis, \(s\) and \(t\) are exogenously given and the sellers choose \(z\) (location) and \(p\) (price) sequentially. Like many other writers, we maintain the assumption that \(t\) is given as it represents consumers’ taste regarding horizontally differentiated products. On the other hand, we fix the locations of sellers to be the end points 0 and 1 of the interval and do not consider the issue of locational choice. \(^2\)

If we also fix \(s\) (as is usually done), then we obtain a pricing game between horizontally differentiated sellers, which leads to some familiar and not-so-familiar results. Many texts in microeconomic theory or industrial organization, such as Mas-Colell et al. (1995) or Tirole (1988), discuss the familiar pricing equilibria under “reasonable” parameter values of \(s\) and \(t\). On the other hand, Wang (2006)

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\(^1\) The fact that the transportation cost is linear in distance is significant for the existence and the properties of equilibria and our later results may not be robust to other specifications. (d’Aspremont et al, 1979) But the linear specification is commonly employed in discussions of linear city models following the lead of Hotelling (1929).

\(^2\) Allowing locations to be at non-boundary points does not change results qualitatively. What we assume is that the incumbent monopolist does not to move the existing product to another location in response to entry of a new product. For linear city and other product differentiation models, strategic choice of locations has been extensively discussed in the literature and we do not go into that direction here. See for example Beath & Katsoulacos (1991) and Gabszewicz & Thisse (1992). Lancaster (1979) suggests some examples where locations may be considered fixed.
and Kim (2006, 2007) explain less well-known characterizations of equilibria under broader ranges of parameter values.

An interesting finding reported in Kim (2006) is that post-entry duopoly equilibrium price may be higher than pre-entry monopoly price. In a similar vein, the fact that increasing number of sellers can lead to higher prices has been noted in such recent papers as Chen & Riordan (2008) and Roessler (2010).³

A natural question then arises as to whether non-price competition can also exhibit a similar phenomenon and this is the question we address in this paper.

It is obvious that many sellers do compete in both price and quality and how entry affects both price and quality is an important research question. (Courtemanche & Carden, 2011) Then the modeling question is how we treat price and quality. In a standard industrial organization framework, it may seem natural to treat price as a short-run variable compared to quality (Tirole, 1988, p.205). Kim (2012) shows that quality degradation and reduction of consumers’ surplus may obtain under a sequential quality and price choice model.

In this paper, however, we focus on the case where quality is the only strategic aspect of competition with prices exogenously fixed. As was argued above, such a setting may be applicable to some important markets where prices are regulated or at least remain relatively stable over an extended period and a few differentiated sellers engage in non-price competition. Examples other than telecommunication services include healthcare industry (Brekke et al., 2011) and airlines industry (Douglas & Miller, 1974) where prices are heavily regulated and “creative industries” (production of movies, music, books and other entertainment products, see Caves, 2000 for details) where a conventional set of contracts prevail for an extended period in pricing practices.

In Section II, we describe our models for the incumbent monopoly and the post-entry game. In Section III, we compare quality levels and consumers’ surplus between the incumbent monopoly and the post-entry game. Section IV concludes.

Appendix contains longer and computational proofs, while some proofs are given in the text.

³ Older papers include Salop (1979), Satterthwaite (1979) and Rosenthal (1980). Surprisingly enough, such perverse effects of entry are also possible even in Cournot models where products are homogeneous. See Frank & Quandt (1963) and Frank (1965) where efforts are made towards removing such perverse effects and Amir & Lambson (2000) where the issue is treated in a more constructive manner.
II. MODEL

The setup of the consumer side follows the standard Hotelling model, which has already given in Introduction. Consumers are uniformly distributed over [0, 1] with net utility \( s - p - t|x - z| \), where \( s \) will be chosen by seller(s) while \( p \) and \( t \) are exogenously given.

Before entry, a monopolist offers a product located at point 0. Assume for simplicity that the per unit production cost is zero, but the seller needs to incur cost \( C(s_0) \) to provide quality \( s_0 \), with \( C'(\cdot) > 0 \) and \( C''(\cdot) > 0 \). For concreteness, let us impose a functional form for \( C(s) \) as an assumption.

**Assumption 1.** \( C(s) = \frac{1}{2}cs^2 \), with \( c > 0 \)

While we will use Assumption 1 to obtain closed-form expressions for various results, key findings should be qualitatively similar with any convex cost function.

An entrant enters with a comparable product at point 1. The new product comes with the quality \( s_1 \), chosen by the entrant with the identical cost function \( C(\cdot) \). We assume that the incumbent accommodates entry and two firms engage in a non-cooperative game with \( s_i \) as their strategic variables. We first characterize optimal choices by the incumbent monopolist before entry.

Let us also introduce an assumption on the size of the “cost” parameters \( c \) and \( t \), which will be in effect throughout the rest of the paper. Under this assumption, a seller’s cost of enhancing quality should not be too high and a consumer’s distaste for non-ideal product should not be too great. This assumption ensures viability of the market by allowing positive profits for seller(s) in equilibrium.

**Assumption 2.** The cost parameters are sufficiently small: \( ct < \frac{1}{2} \)

1. Pre-entry monopoly

Because our incumbent monopolist is located at the left extreme point of the linear
city, its profit maximization involves the decision of whether to cover the whole market, which depends on the level of the exogenously given price $p$. If $p$ is too low, the monopolist finds it not worthwhile to sell to all potential consumers (which requires setting a sufficiently high quality level $s$). We will illustrate this claim more formally below.

A consumer located at $x \in [0,1]$ has net utility $\max(s - tx - p, 0)$ when the monopolist chooses $s$. If net utility enjoyed by the consumer at point 1 is non-negative, i.e. $s - t - p \geq 0$, then demand is 1, while if $s - t - p < 0$, then there exists a marginal consumer at $x_0$ with $s - tx_0 - p = 0$ so demand for the monopolist’s product is $x_0 = \frac{1}{t}(s - p)$. In summary, the demand faced by the monopolist is

$$\left\{ \begin{array}{ll}
\frac{1}{t}(s - p), & s < t + p \\
1, & s \geq t + p
\end{array} \right. \quad (1)$$

and the profit as a function of $s$ is

$$\pi(s) = \left\{ \begin{array}{ll}
\frac{1}{t}p(s - p) - \frac{1}{2}cs^2, & s < t + p \\
p - \frac{1}{2}cs^2, & s \geq t + p
\end{array} \right. \quad (2)$$

If a profit maximizing quality $s$ is less than $t + p$, then the first-order condition dictates that $\frac{1}{t}p - cs = 0$. On the other hand, the monopolist does not have any incentive to raise the quality $s$ above $t + p$ because doing so does not yield any additional revenue (demand is fixed at 1) but incurs additional costs.

Formally, let us express the monopolist’s problem as choosing the optimal amount of demand $x$ with the constraint $x \leq 1$.

$$\max_{x \leq 1} \pi(x) = px - \frac{1}{2}c(p + tx)^2$$

We have $x > 0$ at an optimum because $\pi'(0) = p - ctp = (1 - ct)p > 0$ (by Assumption 2). The Lagrangian function formed in recognition of the constraint $x \leq 1$ is $L = px - \frac{1}{2}c(p + tx)^2 + \lambda(1 - x)s$ and the first-order conditions are (i)
\[ L_x = p - ct(p + tx) - \lambda = 0 \quad \text{and} \quad (ii) \quad L_{\lambda} = 1 - x \geq 0 \ , \ \lambda \geq 0 \ , \] with complementary slackness. Therefore, we have the following characterizations of the optima: If \( x < 1 \), then \( \lambda = 0 \) and \( p - ct(p + tx) = 0 \) or \( \frac{1-ct}{ct^2} p = x < 1 \), so \( p < \frac{ct^2}{1-ct} \). If \( x = 1 \), then \( p - ct(p + t) - \lambda = 0 \) with \( \lambda \geq 0 \), so \( p \geq \frac{ct^2}{1-ct} \).

These results are summarized in the following lemma.

**Lemma II.1**

(a) [partial market monopoly] If price is sufficiently low \( (p < \frac{ct^2}{1-ct}) \), then the incumbent monopolist chooses \( s = \frac{1}{ct} p \) with the resulting profit \( \frac{1-2ct}{2ct^2} p^2 \).

(b) [full market monopoly] If price is sufficiently high \( (p \geq \frac{ct^2}{1-ct}) \), then the incumbent monopolist chooses \( s = p + t \) with the resulting profit \( p - \frac{1}{2} c(p + t)^2 \).

Note that in (a), \( s = \frac{1}{ct} p > 2p \) and the profit is positive by Assumption 2. In (b), \( s = p + t = \left(1 + \frac{t}{p} \right) p \leq \left(1 + t \times \frac{1-ct}{ct^2} \right) p = \frac{1}{ct} p \). The optimal quality \( s \) is a piecewise linear function of \( p \) with two parts where the slope is \( \frac{1}{ct} > 2 \) for low \( p \)'s and the slope is 1 for high \( p \)'s. This holds because at the threshold value of \( p \), the demand becomes inelastic (the market is fully covered).

**2. Post-entry game**

After entry, two sellers each located at points 0 and 1 simultaneously set their quality levels \( s_0 \) and \( s_1 \). The equilibria of this game are characterized in the following lemma.

**Lemma II.2** There are three kinds of Nash equilibria in the post-entry game depending on the value of exogenously given price \( p \).

(1) [duopoly] If price is sufficiently high \( (p > \frac{ct^2}{2(1-2ct)}) \), then in the unique Nash equilibrium the incumbent and the entrant divides the market in half with
identical level of quality $s_0 = s_1 = \frac{p}{2ct}$ with each seller earning the profit
\[
\frac{1}{2} p(1 - \frac{p}{4ct^2}).
\]

(2) [local monopolies] If price is sufficiently low ($p < \frac{ct^2}{2(1-ct)}$), then in the unique Nash equilibrium each seller acts as a local monopolist setting identical level of quality $s_0 = s_1 = \frac{p}{ct}$.

(3) [adjacent sellers] If price is intermediate ($\frac{ct^2}{2(1-ct)} \leq p \leq \frac{ct^2}{2(1-2ct)}$), then there are infinitely many Nash equilibria where the equilibrium quality levels satisfy the relation $s_0 + s_1 = t + 2p$.

The proof is given in Appendix. Classification of potential equilibria into three kinds is similar in spirit to pricing equilibria of Hotelling model where locations are fixed. (Wang, 2006), (Mas-Colell et al., 1995, Exercise 12.C.14)

If price is sufficiently high, then sellers are able to set qualities high enough to induce all consumers to purchase. This is referred to as duopoly in Case (1) above. In this equilibrium, the marginal consumer who is indifferent between the two sellers strictly prefers purchasing from either seller to not purchasing any as her net utility is positive.

In contrast, if price is very low, then it will not be profitable for sellers to offer high qualities, so they are content with being local monopolists. A subset of consumers located in the middle of the city do not purchase from either of them because doing so would yield a negative net utility. This is Case (2) and we refer to this as local monopolies.

Finally, there is an intermediate range of prices where one seller’s served area just touches another seller’s served area. The marginal consumer between them is also indifferent between purchasing and not purchasing. If a seller raises its quality, it can steal some consumers from the other seller but it would lead to an unprofitable competition (price is too low for duopoly to obtain). A seller may attempt to become an isolated local monopolist by lowering its quality, but then the other seller would be happy to expand its served area (price is too high for local monopolies to obtain). This is Case (3) and we refer to it as adjacent sellers.

The reason Case (3) obtains for a (continuum) range of prices is that a seller’s demand schedule is kinked and its marginal revenue curve is discontinuous at the
kink. Equilibrium outcome is indeterminate in the sense that there is a continuum of equilibria and there is no \textit{a priori} reason to select any single equilibrium. But all equilibria in this case satisfy the formula $s_0 + s_1 = t + 2p$, which is a useful characterization.

\section{III. ANALYSIS}

\subsection{1. Comparison of quality levels}

Let us compare quality levels between monopoly and post-entry game. Recollect that there are two cases of monopoly outcomes (from Lemma II.1) depending on a threshold level of price, call it $p_0 \equiv \frac{ct^2}{1-ct}$, and that there are three cases of post-entry outcomes (from Lemma II.2) depending on two threshold levels of price, call them $p_1 \equiv \frac{ct^2}{2(1-ct)}$ and $p_2 \equiv \frac{ct^2}{2(1-2ct)}$. Let us record these notations for future references.

\textbf{Definition 1.} $p_0 \equiv \frac{ct^2}{1-ct}$ \textit{is the threshold price between partial and full market monopoly.}

\textbf{Definition 2.} $p_1 \equiv \frac{ct^2}{2(1-ct)} \textit{is the threshold price between local monopolies and adjacent sellers.}$

\textbf{Definition 3.} $p_2 \equiv \frac{ct^2}{2(1-2ct)} \textit{is the threshold price between adjacent sellers and duopoly.}$

Note that $p_1 = \frac{1}{2}p_0 < p_0$ and $p_1 < p_2$ always but $p_2$ can be greater or less than $p_0$. In fact, it is easy to check

$$p_2 \leq p_0 \Leftrightarrow ct \leq \frac{1}{3}$$

so we can divide our comparisons into two Propositions depending on how $ct$ compares with $1/3$.

Let $s$ denote the incumbent monopolist's quality level and $s_i$ ($i = 0, 1$) denote the two sellers' quality levels after entry.
Proposition 1. Suppose $ct < \frac{1}{3}$ so that $p_1 < p_2 < p_0$.

(0) [partial market monopoly to local monopolies] If $p < p_1$, then the incumbent monopolist does not cover the market and entry leads to local monopolies. Quality levels are not affected by entry, i.e. $s = s_0 = s_1$.

(1) [partial market monopoly to adjacent sellers] If $p_1 < p < p_2$, then the incumbent monopolist does not cover the market and entry can lead to one of infinitely many adjacent sellers equilibria. In all of the post-entry equilibria, the average quality falls compared to the monopoly level, i.e. $s > \frac{s_0 + s_1}{2}$.

(2) [partial market monopoly to duopoly] If $p_2 < p < p_0$, then the incumbent monopolist does not cover the market and entry leads to a unique duopoly equilibrium where quality levels are half of the monopoly level, i.e. $s > s_0 = s_1 = \frac{s}{2}$.

(3) [full market monopoly to duopoly] If $p_0 < p < 4p_2$, then the incumbent monopolist covers the market and entry leads to a unique duopoly equilibrium where quality levels are lower than in monopoly, i.e. $s > s_0 = s_1$.

(4) [full market monopoly to duopoly] If $p > 4p_2$, then the incumbent monopolist covers the market and entry leads to a unique duopoly equilibrium where quality levels are higher than in monopoly, i.e. $s < s_0 = s_1$.

Proposition 2. Suppose $ct > \frac{1}{3}$ so that $p_1 < p_0 < p_2$.

(0) [partial market monopoly to local monopolies] If $p < p_1$, then the incumbent monopolist does not cover the market and entry leads to local monopolies. Quality levels are not affected by entry, i.e. $s = s_0 = s_1$.

(1) [partial market monopoly to adjacent sellers] If $p_1 < p < p_0$, then the incumbent monopolist does not cover the market and entry can lead to one of infinitely many adjacent sellers equilibria. In all of the post-entry equilibria, the average quality falls compared to the monopoly level, i.e. $s > \frac{s_0 + s_1}{2}$.

(2) [full market monopoly to adjacent sellers] If $p_0 < p < p_2$, then the incumbent monopolist covers the market and entry can lead to one of
infinitely many adjacent sellers equilibria. In all of the post-entry equilibria, the average quality falls compared to the monopoly level, i.e. \( s > \frac{s_0 + s_1}{2} \).

(3) [full market monopoly to duopoly] If \( p_2 < p = \frac{2ct^2}{1-2ct} \), then the incumbent monopolist covers the market and entry leads to a unique duopoly equilibrium where quality levels are lower than in monopoly, i.e. \( s > s_0 = s_1 \).

(4) [full market monopoly to duopoly] If \( p > \frac{2ct^2}{1-2ct} \), then the incumbent monopolist covers the market and entry leads to a unique duopoly equilibrium where quality levels are higher than in monopoly, i.e. \( s < s_0 = s_1 \).

Proofs of Proposition 1 and Proposition 2 are given in Appendix.

Entry can lead to a higher level of quality only when exogenously given price is very high – see the item (4) in each Proposition. Mathematically this is because when price is sufficiently high a unit increase in price leads to a unit increase in quality under monopoly \( (s = p + t) \), but the same unit increase in quality leads to a higher increase in quality under duopoly \( (s_0 = s_1 = \frac{1}{2ct} p) \), where \( \frac{1}{2ct} > 1 \).

More intuitively, a sufficiently high price ensures that the incumbent as a monopolist faces a relatively inelastic demand schedule. When price becomes higher, the seller needs to “compensate” buyers with higher quality but with a relatively inelastic demand this need for “compensation” is less urgent. On the other hand, in the post-entry duopoly game, the two sellers are competing for the same market and the demand is more elastic than under monopoly. Higher prices in this case put more pressure on the sellers to provide “compensation” in the form of higher qualities.

One implication of this result is that introducing competition into a price-regulated industry can lead to quality improvement only if price is kept sufficiently high. Unless price is sufficiently high, the pressure of quality competition under duopoly may lead to a lower level of quality in equilibrium. In addition, this happens only when the incumbent monopolist was serving the full market, hence there is no market expansion (which would have created additional consumers’ surplus) from entry.

If price is very low, then the monopolist serves only a small portion of the
potential market and entry creates a separated monopolist, which increases the total consumers’ surplus but quality levels are unaffected in this case.

For other levels of price, quality typically falls after entry. This occurs because quality requires convex cost and entry splits the market between the sellers. If the incumbent monopolist was covering the market only partially, entry causes market expansion as well as quality degradation, hence its effect on consumers’ surplus is ambiguous at this point. We turn to the examination of consumers’ surplus in the next subsection.

2. Comparison of consumers’ surplus

In the previous subsection, we discussed equilibrium levels of quality. We now examine consumers’ surplus between the pre-entry monopoly and the post-entry game. We do not discuss the full social welfare effects mainly because we are going to show that consumers’ surplus typically decreases by entry. Even when consumers’ surplus increases by entry, the incumbent seller’s profit generally decreases by entry and if the fixed entry cost is sufficiently high, then the entry’s welfare benefits may be dubious at best.\(^5\)

There are two different monopoly outcomes and three different post-entry outcomes. For our purpose, we can ignore one of the post-entry outcomes – where price is so low that the two sellers’ markets do not overlap at all and each acts as a local monopolist. In this case, entry simply creates another independent market, which is obviously a socially efficient move. Hence, we drop references to item (0) of Propositions 1 and 2.

Let us first characterize consumers’ surplus for various cases. Let \(CS_1\) and \(CS_2\) denote the consumers’ surplus before and after entry, respectively.

**Lemma III.1** [monopoly] If \(p < p_0\), then \(CS_1 = \frac{(1-ct)^2}{2e^2 t^3} p^2\). If \(p > p_0\), then \(CS_1 = \frac{1}{2} s = \frac{1}{2} (p + t)\).

Proof is straightforward and omitted.

\(^5\) Although we do not report the details for brevity, we can show social welfare decreases for several parameter ranges, even without accounting for fixed entry costs.
Lemma III.2 [adjacent sellers] In an adjacent sellers equilibrium, $CS_2$ depends on how two sellers divide the market. When one seller’s market share is $\mu$, we have

$$\frac{1}{4}t \leq CS_2 = \left[ \left( \mu - \frac{1}{2} \right)^2 + \frac{1}{4} \right] t \leq \frac{1}{2}t$$

so that $CS_2$ is minimized when the market is shared equally and is maximized when one seller gets the whole market.

Proof: Entry may lead to one of an infinitely many different equilibria with $s_0 + s_1 = 2p + t$. If we fix $s_0 = \sigma$, then the incumbent’s demand (and its market share) is $\mu = \frac{1}{t}(\sigma - p)$ so that $\sigma - p = \mu t$. Hence

$$CS_2 = \frac{1}{2t} (\sigma - p)^2 + \frac{1}{2t} [t - (\sigma - p)]^2 = \frac{1}{2t} [(\mu t)^2 + (t - \mu t)^2] = \left[ \left( \mu - \frac{1}{2} \right)^2 + \frac{1}{4} \right] t$$

(See Figure 1 below.) The same formulas obtain when the entrant’s market share is $\mu$. Characterization of the minimum and the maximum follows from $0 \leq \mu \leq 1$. □

<Figure 1> Lemma III.2 $CS = (A) + (B)$

![Diagram](image-url)
The lemma above characterized $CS_2$ in terms of a seller’s market share. The following corollary gives an alternative characterization in terms of Herfindahl-Hirschman Index (HHI). Since we have only two sellers, HHI is $H^2 = \mu^2 + (1 - \mu)^2$, when one seller’s market share is $\mu$.

**Corollary III.3 [adjacent sellers]** If $p_1 < p < p_2$, $CS_2 = \frac{1}{2} tH^2$ where $\frac{1}{2} \leq H^2 \leq 1$.

**Proof:** $CS_2 = \frac{1}{2t} [\mu^2 + (t - \mu)^2] = \frac{1}{2t} t[\mu^2 + (1 - \mu)^2] = \frac{1}{2} tH^2$. When $\mu = \frac{1}{2}, H^2 = \frac{1}{2}$. When $\mu = 1, H^2 = 1$. ■

**Lemma III.4 [duopoly]** If $p > p_2$, then $CS_2 = \frac{1-2ct}{2ct} p - \frac{1}{4} t$.

Proof is straightforward and omitted.

We can now make comparisons. Let us first consider the case covered by Proposition 1.

**Proposition 3.** Suppose $ct < \frac{1}{3}$.

1. **[partial market monopoly to adjacent sellers]** When price is low ($p_1 < p < p_2$) so that entry transforms the partial market monopoly into an adjacent sellers equilibrium, consumers’ surplus increases if price is sufficiently low, i.e. $p < \frac{ct^2}{1-ct} H$, where $H$ is the square root of Herfindahl-Hirschman Index. The more unevenly the market is divided, the more likely it is that entry increases consumers’ surplus.

2. **[partial market monopoly to duopoly]** When price is intermediate ($p_2 < p < p_0$) so that entry transforms the partial market monopoly into duopoly, consumers’ surplus decreases.

3. **[full market monopoly to duopoly]** When price is high ($p > p_0$) so that entry transforms the full market monopoly into duopoly, consumers’ surplus may decrease for a range of lower prices but increases for sufficiently high prices.

While proof is given in Appendix, a few remarks are in order. Entry has two potentially conflicting effects on consumers’ surplus. If entry expands the market, *i.e.* if previously unserved consumers are served after entry, then new surpluses are
created. This effect can be expected only when the incumbent monopolist covers the market partially. On the other hand, entry may lead to higher or lower levels of quality, which is directly related to whether consumers’ surplus increases or decreases.

In case (1), the market is expanded by entry, which will increase consumers’ surplus. However, the average quality of the two sellers is lower than the monopolist’s quality (see item (1) in Proposition 1). When the market is shared relatively evenly between the sellers, it means both sellers choose relatively low levels of quality, which may lead to reduction of consumers’ surplus. For example, if the two sellers share the market equally, then consumers’ surplus decreases by entry.

On the other hand, consumers’ surplus is maximized in an adjacent sellers equilibrium where one of the sellers cover the whole market. This situation is different from monopoly, because if the single seller sets the monopoly quality, then entry will occur. In order to prevent entry, the monopolist (whether the incumbent or the entrant) needs to set a sufficiently high level of quality, which leads to a higher level of consumers’ surplus.

Therefore, a somewhat surprising implication is that if the policy maker is determined to keep the price low, then consumers’ surplus can be maximized by threatening to allow (but not actually allowing) entry. Even when entry occurs, it is better (in terms of consumers’ surplus) to keep an uneven market distribution between the sellers.

In case (2), although entry expands the market, the duopoly competition leads to a substantial reduction in quality levels so that consumers’ surplus decreases.

Finally in case (3), there is no market expansion effect so the only way entry can benefit consumers is to ensure quality levels to rise, which happens only if a sufficiently high price is maintained.

We now turn to the case where \( ct > \frac{1}{3} \). While the findings are generally similar to the previous Proposition, the condition \( ct < \frac{1}{3} \) has been used in several places in the proof of Proposition 3, so we do need to work out the results for \( ct > \frac{1}{3} \) separately.

**Proposition 4.** Suppose \( ct > \frac{1}{3} \) (We also use Assumption 2, so \( ct < \frac{1}{2} \))

(1) [partial market monopoly to adjacent sellers] When price is low \((p < p_0)\)
so that entry transforms the partial market monopoly into an adjacent sellers equilibrium, consumers’ surplus increases if price is sufficiently low, i.e. \( p < \frac{ct^2}{1-ct} H \), where \( H \) is the square root of Herfindahl-Hirschman Index. The more unevenly the market is divided, the more likely it is that entry increases consumers’ surplus.

(2) [full market monopoly to adjacent sellers] When price is intermediate \((p_0 < p < p_2)\) so that entry transforms the full market monopoly into an adjacent sellers equilibrium, consumers’ surplus decreases.

(3) [full market monopoly to duopoly] When price is high \((p > p_2)\) so that entry transforms the full market monopoly into duopoly, consumers’ surplus decreases.

Differences from Proposition 3 are as follows. First, in case (1) the portion of parameter values where consumers’ surplus may decrease is relatively larger (compare Figures 2 and 3 in Appendix). It still remains true that an uneven distribution of market shares means a higher consumers’ surplus. Second, in cases (2) and (3) without the market expansion effects, quality alone cannot increase consumers’ surplus even if price is kept sufficiently high. This is because in this Proposition the cost parameters are relative large.

IV. CONCLUSION

In this paper, we considered a simple variant of Hotelling model (1929) where the conventional choice variables \((p\) and locations\) as well as the conventional parameters \((t)\) are fixed but quality (gross surplus from the product) is a choice variable.

By characterizing monopolist’s quality choices and the post-entry equilibrium choices of quality in detail, we showed that for a surprisingly broad range of parameter values post-entry quality levels can be lower and consumers’ surplus can be reduced by entry.

While these results have been obtained using a particular functional form for cost of setting qualities (Assumption 1) and also under the simplifying assumption of locations being fixed at the boundary points of the linear city, it is easy to see that results will be qualitatively similar for any convex cost functions and for non-
boundary locations (as long as the incumbent does not change its location in response to entry). Similar results can be obtained even for a circle model (Salop, 1979), where the crucial parameter conditions will hinge upon the cases where a seller’s market overlaps with a neighboring seller’s market.

Although our model is very simple so that a care should be taken in making references to real-world applications, we can draw some cautious implications as follows. First, we should not consider entry per se to be beneficial to consumers without careful examinations, especially when products are horizontally differentiated. Second, a policy maker needs to be cautious in assessing any policy’s potential effects, be it setting of a regulated price or permission of a new entrant into the market, for quality choices may be affected (in adverse and somewhat unexpected ways) by such decisions.

REFERENCES

Entry and Quality Competition in Hotelling Model with Exogenous Prices


APPENDIX: PROOFS

Proof of Lemma II.2

Case (1): Let us first consider the case of a standard duopoly equilibrium. If sellers choose $s_0$ and $s_1$ and the exogenous price is $p$, then the location $y^H$ of the marginal consumer who is indifferent between the two sellers is determined by

$$s_0 - p - t y^H = s_1 - p - t(1 - y^H) \Rightarrow y^H = \frac{1}{2} + \frac{s_0 - s_1}{2t}$$

Profits are

$$\pi_0(s_0, s_1) = p y_0^H - \frac{1}{2} c s_0^2 = p \left(\frac{1}{2} + \frac{s_0 - s_1}{2t}\right) - \frac{1}{2} c s_0^2$$

$$\pi_1(s_0, s_1) = p(1 - y^H) = p(1 - y) - \frac{1}{2} c s_1^2 = p \left(\frac{1}{2} - \frac{s_0 - s_1}{2t}\right) - \frac{1}{2} c s_1^2$$

Each seller tries to maximize own profit by solving either of the following two first-order conditions:

$$\frac{\partial \pi_0}{\partial s_0} = \frac{p}{2t} - c s_0 = 0, \quad \frac{\partial \pi_1}{\partial s_1} = \frac{p}{2t} - c s_1 = 0$$

and it turns out that the optimal choices are

$$s_0 = s_1 = \frac{p}{2ct}$$

At this equilibrium, net utility enjoyed by the marginal consumer at $y^H = \frac{1}{2}$ is

$$s_0 - p - t y^H = \frac{p}{2ct} - p - \frac{1}{2} t = \left(\frac{1}{2ct} - 1\right) p - \frac{1}{2} t$$

---

6 This proof closely follows Wang’s (2006) arguments.

7 It is trivial to check that the second-order conditions hold.
which must be positive, so we need \( p > \frac{\frac{1}{t} \cdot \frac{1}{2ct}}{\frac{1}{2(1-ct)}} = \frac{ct^2}{2(1-ct)} \). Hence we obtain Case (1).

Case (2): Let us now consider the case of local monopolies. The two sellers need not act strategically because their respective markets do not overlap and there is a middle region of consumers who do not purchase from either of them. In order to use Lemma II.1 (a) [partial market monopoly], assume \( p \) is sufficiently low. Each seller then will choose \( s_0 = s_1 = \frac{1}{ct} p \), with the locations of their respective marginal consumer at \( y_0 = \frac{1}{t} (s_0 - p) = \frac{1}{t} \left( \frac{1}{ct} - 1 \right) p \) and \( y_1 = 1 - \frac{1}{t} (s_1 - p) = 1 - \frac{1}{t} \left( \frac{1}{ct} - 1 \right) p \). We need to have \( y_0 < y_1 \), so \( p < \frac{ct^2}{2(1-ct)} \) is the needed condition.

Case (3): The above characterizations leave a range of prices. At the quality levels satisfying the first order conditions in Case (1), the marginal consumer does not enjoy positive net utility so may choose not to purchase the product. We need to consider individual seller’s best response, given an arbitrary choice by its competitor. We shall focus on the position of the incumbent. (The entrant’s position can be argued along similar lines.)

We can quickly restrict the possible ranges of \( s_1 \) as follows. If \( s_1 < p \), then no consumer finds it worthwhile to purchase from the entrant, so the incumbent can safely act as a monopolist [Case (2)]. On the other hand, if \( s_1 > p + t \), then the entrant’s quality is so high that all consumers would consider purchasing from the entrant, hence the incumbent must engage in duopoly competition [Case (1)].

Therefore, suppose \( p \leq s_1 \leq p + t \). Then the location \( y_1^M \) of the marginal consumer for the entrant is found by \( s_1 - p - t(1 - y_1^M) = 0 \) or \( y_1^M = 1 - \frac{1}{t} (s_1 - p) \). Now, the minimum quality \( \underline{s} \) that the incumbent can charge to the entrant’s marginal consumer is found by setting her net utility from the incumbent to be zero, or \( \underline{s} - p - ty_1^M = 0 \), which leads to \( \underline{s} = p + ty_1^M = p + t \left( 1 - \frac{1}{t} (s_1 - p) \right) = 2p + t - s_1 \). Notice here that we have just obtained the relation \( \underline{s} + s_1 = 2p + t \). At \( s_0 = \underline{s} \), the incumbent serves the exact remainder of the entrant’s market, so the incumbent’s demand is \( y_0^M = 1 - y_1^M = \frac{1}{t} (s_1 - p) \).

We could write out a detailed argument, along the lines given in Wang (2006), that clarifies the exact parameter conditions required for these equilibria. However, obtaining the condition \( s_0 + s_1 = t + 2p \) is sufficient for our purposes in this paper.
Proof of Proposition 1

(1) \( s = s_0 = s_1 = \frac{1}{ct}p \).

(2) \( s - \frac{s_0 + s_1}{2} = \frac{1}{ct} - \left( \frac{t}{2} + p \right) = \left( \frac{1}{ct} - 1 \right) p - \frac{t}{2} > \left( \frac{1-ct}{ct} \right) p_1 - \frac{t}{2} = 0. \)

(3) \( s = \frac{1}{ct}p > s_0 = s_1 = \frac{1}{2ct}p. \)

(4) \( s - s_0 = p + t - \frac{1}{2ct}p = \left( 1 - \frac{1}{2ct} \right) p + t > \left( 1 - \frac{1}{2ct} \right) \left( \frac{2ct^2}{1-2ct} \right) + t = 0. \)

The inequality holds because \( 1 - \frac{1}{2ct} < 0 \) and \( p < \frac{2ct^2}{1-2ct}. \)

(5) \( s_0 - s = \frac{1}{2ct}p - (p + t) = \left( \frac{1}{2ct} - 1 \right) p - t > \left( \frac{1}{2ct} - 1 \right) \left( \frac{2ct^2}{1-2ct} \right) + t = 0. \) ■

Proof of Proposition 2

(1) \( s = s_0 = s_1 = \frac{1}{ct}p. \)

(2) \( s - \frac{s_0 + s_1}{2} = \frac{1}{ct} - \left( \frac{t}{2} + p \right) = \left( \frac{1}{ct} - 1 \right) p - \frac{t}{2} > \left( \frac{1-ct}{ct} \right) p_1 - \frac{t}{2} = 0. \)

(3) \( s - \frac{s_0 + s_1}{2} = p + t - \left( \frac{t}{2} + p \right) = \frac{t}{2} > 0. \)

(4) \( s - s_0 = p + t - \frac{1}{2ct}p = \left( 1 - \frac{1}{2ct} \right) p + t > \left( 1 - \frac{1}{2ct} \right) \left( \frac{2ct^2}{1-2ct} \right) + t = 0. \)

The inequality holds because \( 1 - \frac{1}{2ct} < 0 \) and \( p < \frac{2ct^2}{1-2ct}. \)

(5) \( s_0 - s = \frac{1}{2ct}p - (p + t) = \left( \frac{1}{2ct} - 1 \right) p - t > \left( \frac{1}{2ct} - 1 \right) \left( \frac{2ct^2}{1-2ct} \right) + t = 0. \) ■

Proof of Proposition 3

(1) From Lemma III.1, \( CS_1 = \frac{(1-ct)^x_2}{2c^2t^3}p^2 \) and from Corollary III.3, \( CS_2 = \frac{1}{2}tH^2. \)

\[ \Delta CS = CS_2 - CS_1 = \frac{1}{2}tH^2 - \frac{(1-ct)^2}{2c^2t^3}p^2 > 0 \iff p^2 < \frac{c^2t^4}{(1-ct)^2}H^2 \]

\[ \iff p < \frac{ct^2}{1-ct}H \]
Let \( p(H) \equiv \frac{ct^2}{1-ct} H \). Since \( \frac{1}{2} \leq H^2 \leq 1 \), we have

\[
p_1 = \frac{ct^2}{2(1-ct)} < p\left(\frac{1}{\sqrt{2}}\right) = \frac{ct^2}{\sqrt{2}(1-ct)} \leq p(H) \leq p(1) = \frac{ct^2}{1-ct} = p_0
\]

The threshold \( p(H) \) falls between \( p_1 \) and \( p_0 \), but it may fall below or above \( p_2 \). (See Figure 2.)

\(<\text{Figure 2}\> \text{ Proposition 3(1) } \Delta CS > 0 \text{ for } p < p(H), \text{ where } p(H) \text{ falls between } p_1 \text{ and } p_0.\)

\[
p < p(H), \quad H = \frac{1}{\sqrt{2}}
\]

\[
\begin{align*}
\hat{p}_1 &= \frac{ct^2}{2(1-ct)} \\
\hat{p}_2 &= \frac{ct^2}{2(1-2ct)} \\
\hat{p}_0 &= \frac{ct^2}{1-ct}
\end{align*}
\]

(2) From Lemma III.1, \( CS_1 = \frac{(1-ct)^2}{2c^2t^3} p^2 \) and from Lemma III.4, \( CS_2 = \frac{1-2ct}{2ct} p - \frac{1}{4} t. \)

\[
\Delta CS = \left(\frac{1}{2ct} - 1\right) p - \frac{t}{4} - \frac{1}{2t} \left(\frac{1}{ct} - 1\right)^2 p^2
\]
If we plot $\Delta CS$ as function of $p$, then it is a quadratic parabola with the global maximum achieved at $p = \frac{ct^2(1-2ct)}{2(1-ct)^2}$. If we compute the value of $\Delta CS$ at that maximum it turns out that

$$\Delta CS = \frac{1 - 2ct}{2ct} \times \frac{ct^2(1-2ct)}{2(1-ct)^2} - \frac{1}{4t} - \frac{1}{2t} \left(\frac{(1-ct)^2}{c^2t^2}\right) \times \frac{c^2t^4(1-2ct)^2}{4(1-ct)^2}$$

$$= \frac{t}{8(1-ct)^2} \times (2c^2t^2 - 1) < 0$$

The negative sign can be determined because of the condition $ct < \frac{1}{3}$.

(3) From Lemma III.1, $CS_1 = \frac{1}{2} (p + t)$ and from Lemma III.4, $CS_2 = \frac{1-2ct}{2ct} p - \frac{1}{4t}$.

$$\Delta CS = \left(\frac{1}{2ct} - 1\right) p - \frac{t}{4} - \frac{1}{2} (p + t) = \frac{1 - 3ct}{2ct} p - \frac{3}{4} t$$

Hence the sign of change is determined by $\hat{p}$ where $\frac{1 - 3ct}{2ct} \hat{p} - \frac{3}{4} t = 0$ or

$$\hat{p} \equiv \frac{3ct^2}{2(1-3ct)}$$

We can easily check $p_0 < \hat{p}$ by

$$\hat{p} - p_0 = \frac{3ct^2}{2(1-3ct)} - \frac{ct^2}{1-ct} = \frac{3(1-ct) - 2(1-3ct)}{2(1-3ct)(1-ct)} ct^2$$

$$= \frac{1 + 3ct}{2(1-3ct)(1-ct)} ct^2 > 0$$
Hence, there exists a range of prices, $p_0 < p < \hat{p}$, where $\Delta CS < 0$. On the other hand, if $p > \hat{p}$, then $\Delta CS > 0$. The value $\hat{p}$ may be higher or lower than $$\frac{2ct^2}{1-2ct}$$ which is the threshold for quality increases by entry. In other words, quality increase itself need not guarantee an increase in consumers’ surplus.

**Proof of Proposition 4**

(1) We can repeat proof of (1) in Proposition 3, the only difference being that $p(H)$ falls entirely within $[p_1, p_0]$. (See Figure 3.)

\[ p < p(H), H = \frac{1}{\sqrt{2}} \]

\[ p < p(H), H = 1 \]

\[ p_1 = \frac{ct^2}{2(1 - ct)} \quad p_0 = \frac{ct^2}{1 - ct} \quad p_2 = \frac{ct^2}{2(1 - 2ct)} \]

(2) $CS_1 = \frac{1}{2} (p + t), CS_2 = \frac{1}{2} tH^2$

$$\Delta CS = CS_2 - CS_1 = \frac{1}{2} tH^2 - \frac{1}{2} (p + t)$$
ΔCS is maximized when $H$ is maximal at $H^2 = \frac{1}{2}$ and $p$ is minimal at $p = p_0 = \frac{ct^2}{1-ct}$, hence

$$\Delta CS \leq \max(\Delta CS) = \frac{1}{2} t - \frac{1}{2} \left( \frac{ct^2}{1-ct} + t \right) = -\frac{ct^2}{2(1-ct)} < 0$$

(3) From Lemma III.1, $CS_1 = \frac{1}{2} (p + t)$ and from Lemma III.4, $CS_2 = \frac{1-2ct}{2ct} p - \frac{1}{4} t$.

$$\Delta CS = \left( \frac{1}{2ct} - 1 \right) \frac{1}{4} - \frac{1}{2} (p + t) = \frac{1-3ct}{2ct} p - \frac{3}{4} t < 0$$

where the inequality follows from $ct < \frac{1}{3}$. ■